Symulacja w Badaniach i Rozwoju
Vol. 5, No. 4/2014

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Dynamics of a Rolling Railway Wheel: Corrugation, Polygonalization and Flat Spots

1 Introduction

The quality of the surfaces of wheels and rails is an important factor forthe safety and comfort of railway travel as well as for the noise emission.In particular, in densely inhabited regions, the second factor becomes moreand more important. The main issues in this regard are corrugations on rails and wheels and polyganalization of wheels. Corrugations, also knownas *slip waves*, are short length variations of a wheel's radius, respectively a rail's height, with a wavelength of about 3-10 cm. For reference see e.g. [3, 4].Polygonalization, on the other hand, is characterized by a small number ofminima and maxima of the radius around the circumference of a wheel. Assuminga radius of 0.5m and 3 to 12 humps of the radius vs angle function, the wavelength is between 0.25cm and m.

In the past 20 years, polygonalization was associated with high-speed trains. It was investigated in the case of the ICE of DB, TGV of SNCF as well as the Pendolino of several railway authorities.In the case of classical trains at speeds well below 200km/h, the focus was on corrugation (wavelenghs between 5 and 10 cm) rather thenpolygonalization (wavelengths of 1/3 to 1/10 of the wheel's circumference).

Recently, however, on vehicles of low and moderate speed, pologonalization was observed. One of the working hypotheses is that the effect in this case is initiated by flat spots, and later growing on its own.

Fig. 1. Photographs of running surfaceswith flat spots

Rys. 1. Fotografie powierzchni tocznej kół kolejowych z miejscami płaskimi

Another hypothesis is connected with too high residual compressionalcircumferential stress in the wheel-tyre.

2 Aim of the paper

In this paper, we collect some recent results of measured patterns on worn wheels of different trains. Next, we approximate the data by suitable functions,such as trigonometrical polynomials or periodic splines.For uniform guiding motion, the trajectory of the contact point and the resulting vertical accelerations, creepages and frictional forces are evaluated, first in the case of a rigid wheel, then in the case of elastic and visco-elastic contact, Fig. 2.

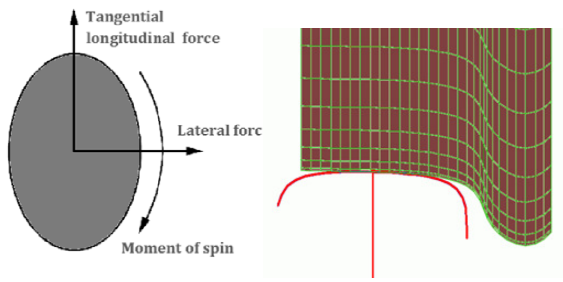

Fig. 2. Contact area and contact forces, cf. [11]
Rys. 2. Strefa kontaktu i siły kontaktowe, por. [11]

The resulting energy dissipation due to friction and percussive effects is calculated, and on that basis the further pattern formation is simulated.

In particular, imperfections in the form of polygonalization, corrugation and flat spots are investigated, like in ref. [5, 13] and [22].

3 Results of Measurements and Initial Data

In this section we collect and describe some examples of worn wheels. We concentrate on instances of flat spots and of full grown'polygonal' shapes.

In figure 1 and figure 3 typical flats are shown. The reasons of the localized removal of material from the surface are schematically presented in figure 3 together with an additional view of the affected region.

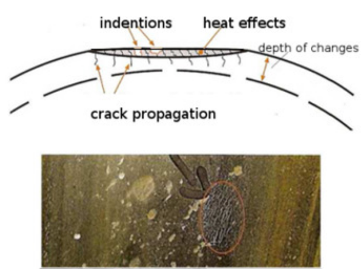

Fig. 3. From experimental investigation to model assumptions
Rys. 3. Od badań doświadczalnych do założeń modelowych

During the further exploitation of the running gear with a flat spot, a largernumber of damaged regions may occur in the form of moreor less regular patterns, such as obtained in figure 4, where the radius function exhibits nine local minima and nine maxima.

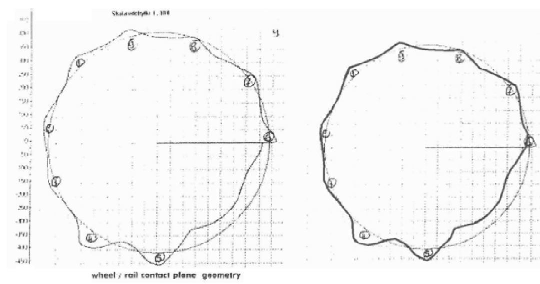

Fig. 4. Original data sheet (left) and approximation (right)
Rys. 4. Arkusz pomiarowy i aproksymacja

It should be pointed out that the presented radius function is an average over the wheel's tread. A distribution over the width, alongthe axle, is shown for two wheels of the same wheelset in Fig. 5

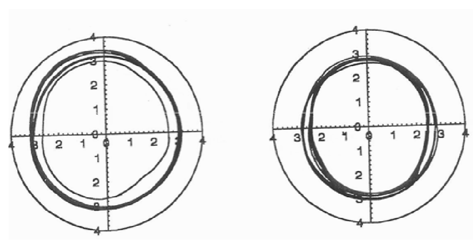

Fig. 5. Radius functions for several planes, left and right wheels of the same set
Rys. 5. Przekroje lewego i prawego koła jednego zestawu w różnych płaszczyznach

In a functional representation, a nonogonial radius function is given in figure 6. The figure shows an original data sheet obtained from measurements at the laboratory of the Railway Institute IK (formerlyCNTK) in 2004.

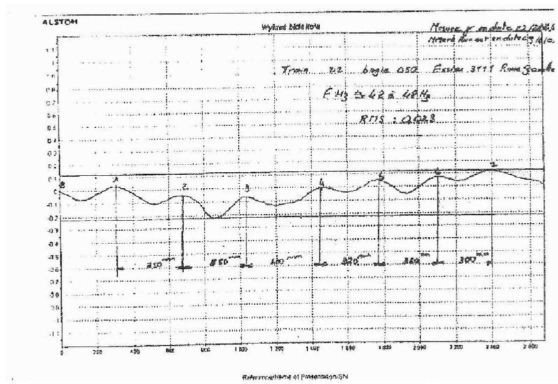

Fig. 6. Original sheet of radius measurement, dependence on angle

Rys. 6. Arkusz pomiaru promienia w zależności od kąta

4 Modeling

A first attempt at the assessment of consequences of the shown deviations from the ideal constant wheel radius is based solely on geometric considerations. The measurred, or assumed actual radius function is temporarily considered invariant, regardless what forces may act on the wheel. Let us consider that the wheel center travels at constant speed along an ideal rail, thewheel revolving at a spin yielding zero creepage in the contact point. Then already at comparatively low speeds an importand effect may be observed.

In fact, the point representing the geometrical contact moves forth and back around the actual longitutinalposition of the wheel center. Thus we have to deal with a normalforce on the rail, which travels at a speed varying around the traveling speed (Fig. 11). Depending on the amplitude of the geometrical imperfection, very large surges of speed, even jumps, may occur. Critical regions of speed for dimensionless speed and frequency, which are shown in Fig, 7, with respect to wave effects in the Bernoulli-Eulerand Rayleighbeam models of the track, may be crossed, see [6, 9].

In a number of previous papers devoted to the dynamics of periodical structures under a moving load, various types of oscillations were studied, [6]. The problem of waves generated by a force traveling at a periodically changing velocityis so far not solved in a satisfactory way.

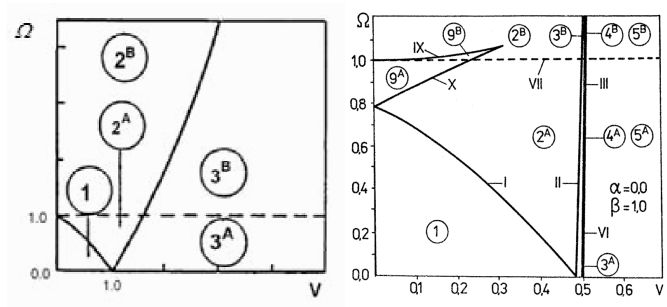

Fig. 7. Regions with qualitatively different solutions in the frequency–velocity plane for the Bernoulli-Euler and Rayleigh beam models

Rys. 7. Obszary jakościowo różnych rozwiązań na płaszczyźnie częstość–prędkość dla modeli belek Bernoulliego-Eulera i Rayleigha

A second consequence of a nonconstant radius function is the variation of dynamical quantities such as tangential and normal contact forces. In fact, if the deformations due to compliance of the track and the elasticity of the contact partners are disregarded, the wheel center has to follow a unilateral constraint, whichin turn results in considerable normal forces. At the same time, theideal rolling condition leads to variations of the angular velocity of thewheel's revolutions, which requires a torque, i.e. a tangential forcein the point of contact.

In a more realistic approach, tangential forces are limited by the friction law, hence torque leads a relative motion of thecontact partners and hence to varying rotational speed. The same forces lead to acceleration/deceleration of the traveling motion. A simple model of a single wheel is presented in Fig. 7. The behavior of models of railway vehicles of this type was studied with special regard to the frictional power dissipated in the contact region, [2, 7, 10], [14–18]. Assuming that the speed of wear is a function of the power dissipation, it is possible to calculate the depth of abrasion starting from a given initial radius imperfection, such as a flat spot.

Here also nonlinearwheel/rail interaction models should be mentioned. For example, rigid and elastic contact models were discussed in [19], elastic and perfect plastic impact models were studied in [8], and generalized in [20]with a discussion of frictional wear as source of surface pattern development in rolling contact. In the approach made by Kowalska [20], a two-dimensional problem with jumps of wheel-rail contact and inelastic impacts was investigated.

5 Geometrical aspects of the contact

Before we will come to simulations on the basis of more realistic data and models of motion, let us study some geometrical aspects of the wheel-rail contact.

The wheel’s circumference, in reference configuration, is defined by

$$r(\varphi; a_w, \omega_w) = R_0 + a_w \sin(\omega_w \varphi) \,,$$
$$x_0(\varphi) = r(\varphi; a_w, \omega_w) \cos(\varphi) \,, \qquad (1)$$
$$y_0(\varphi) = r(\varphi; a_w, \omega_w) \sin(\varphi).$$

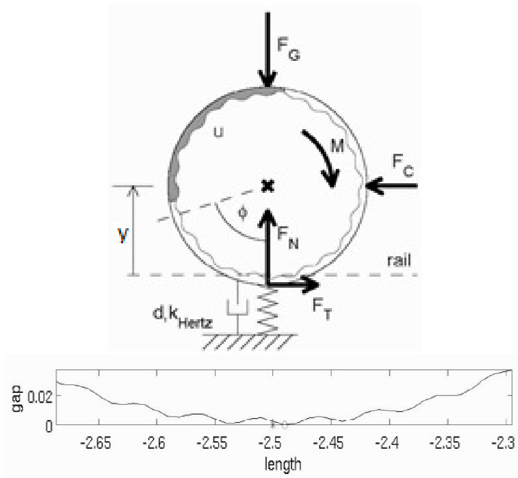

Fig. 8. Elementary roller model (top) and gap between rail and wheel (bottom)
Rys. 8. Prosty model koła (góra) oraz szczelina między kołem i szyną (dół)

In this idealized case of assumed periodicity of the corrugation pattern, some analytical considerations are possible, without the need to apply numerical methods. For this article, we the following parameters:

rail length: L=10 m	wheel radius: R_0=0.46 m;
corrugation wave length: λ_r=0.05 m	number of waves: ω_w =5 wave length: λ_w=0.59 m
corrugation amplitude: a_r =1.25E-4m	wave amplitude: a_w=1E-4 m

In a study of wheel imperfections in contact with a realistic rail, however, we have the problem of two corrugated surfaces.Hence we need to determine, for each angle and each horizontal position of the wheel center, the lowest vertical position such that contact occurs. This can be done by examinating the gap between the contact partners, looking for the minimizer, cf. bottom part of Fig. 8. A very essential issue is the uniqueness of the point of contact, i.e. the minimizer of the gap function. In general, uniqueness holds only up a certain critical level of amplitudes, which in turn depends on the wave numbers of the patterns.

In the case of an ideal track, multiple contact is impossible as long as the wheel remains convex. The limit case occurs when the curvature changes sign.

For the curvatures of an ideal wheel and that of the corrugation imposed, one obtains:

$$\kappa_{ideal} = \frac{1}{R_0}\,; \quad \kappa_{min_{\mathrm{corr}}} = -a_w \frac{\omega_w^2 + 1}{R_0^2}\,. \tag{2}$$

Consequently, the condition for convexity, i.e. nonnegativity of the total curvature, reads

$$a_w \leq \frac{R_0}{\omega_w^2 + 1}\,. \tag{3}$$

For example, if the given wave number is 37, the critical amplitude of corrugations is 0.34mm. Above that level bouncing of the wheel occurs. In the case of both partners featuring corrugations, the uniqueness or multiplicity of contact points depends on the relative position of the bodies with respect to each other.

We confirmed that the above rather rough estimate is consistent with the full analytical evaluation of the curvature and neglecting higher order terms as well as with a straightforward numerical test of convexity.

For both surfaces corrugated, in a similar way a sufficient condition for uniqueness of the geometrical contact point may be derived. We impose the condition that the gap is a convex function, and we estimate the second derivative of the gap function by the difference of minimum wheel curvature minus maximum rail curvature. For this difference positive, no multiple contact may occur.

After calculation we can conclude that for given wave lengths of the perturbation, the admissible range for the amplitudes is a triangle. For given amplitudes, the domain for frequencies is bounded by an ellipse. For an ideal rail and a wheel radius of 0.46m, the critical amplitude limit is 1.84cm, hence uniqueness is not an issue. A flat spot of an extension of 3 cm corresponds to a single trough of a sinosoidal radius profile with a wave number of about fifty and an amplitude of 0.18mm.

Obviously, if the above conditions are violated, a smooth rolling contact is geometrically impossible. On the other hand, if curvatures are in the order which is necessary for a continuous contact, still bouncing may be caused by lift-off due to inertia forces, see e.g. [1, 21].

Let us consider, on an ideal track, the motionat a traveling speed *V*of a polygonalized wheel with nominal radius R_0, a moderate wave number and amplitude of its deviation $\Delta R(\varphi)=r(\varphi)-R_0= a_w \sin(\omega_w\varphi)$. Now, neglecting creepage, the trajectory of the wheel center is in very good approximation given by:

$$x_c(t) = Vt; \quad y_c(t) = R_0 + a_w \cos\left(\omega_w \frac{x_c(t)}{2\pi R_0}\right). \tag{4}$$

Hence, we have a harmonic vertical motion of a mass of around 250kg with a frequency of $\omega_w V/R_0$. The corresponding accelerations are the square of that coefficient multiplied by the amplitude of the out-of-roundness. Lift-offoccursfirst at the local maxima of the trajectory, when the following relation (5) is fulfilled:

$$a_w m_w \left(\frac{\omega_w V}{2\pi R_0}\right)^2 \leq F_w\ . \tag{5}$$

Here F_w is the external load per wheel, assumed to be constant and independent of the wheel center oscillation, and m_w is the mass attached to the wheel center.

6 Compliant models

The pure geometric approach leads to a mechanical system with kinematic constraints, which are enforced by Lagrange multipliers. The latter can not be calculated from the position coordinates and velocities alone, they result from the solution of the system of equations of motion.As an alternative, a stiff elastic system may be formulated, for which violation of the contact condition – in the form of a penetration of the undeformed shapes of the contact partners – is kept small by a normal force, e.g. Winklerian or Herzian. Both approaches allow a unilateral treatment, i.e., positive vertical gaps may be allowed. In time intervals, where this occurs, we have a free flight of the wheel – which ends with an impact.

Later on we will assume a load of around 20t on the considered wheel, which has a radius of about half a meter. A typical contact patch has a length of around 1 to 2cm, which is compatible with an elastic approach of around one 0.1mm. Indeed, it holds

$$\Delta_y = R_0 \sqrt{1 - \frac{l^2}{4R_0^2}} \ . \tag{6}$$

For calculations we chose l =0.02m, R0=0.46m.

In Fig. 9, on the left side, we present the unilateral contact between a wheel with flat spot (exaggerated magnitude of out-of-the-roundness) and a straight rail. On the right part, the straight line corresponds to a linear contact spring, the convex curve showsHertzian contact, assuming a mean curvature of 1/R0 in the contact region.

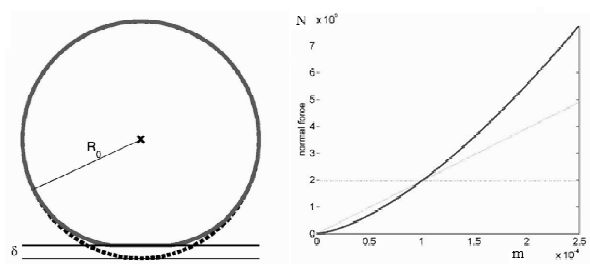

Fig. 9. Wheel-rail contact near a flat spot (left)and normal force vs elastic approach (right)

Rys. 9. Kontakt koło-szyna w obecności płaskiego miejsca (lewa strona) i siła normalna w zależności od głębokości przenikania (po prawej)

In order to find a reasonable damping, let us bounce a wheel vertically onto a perfect (straight) rail. We show results for d =0, d =1e^5, d =4e^5, d =1e^6(in Ns/m). We regard the result for d =4e^{5}as most realistic, see Fig. 10. We have to remember that the contact stiffness according to Hertz depends on the difference between curvatures. So, further refinement is possible here.

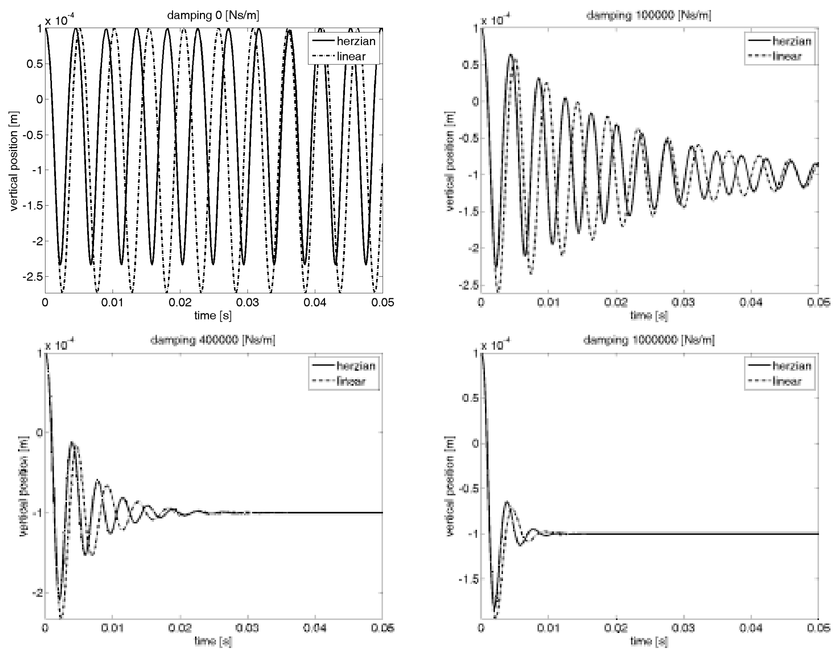

Fig. 10. Vertical displacements for several damping coefficients,
Hertzian and linear contact models

Rys. 10. Przemieszczenia pionowe dla kilku wybranych współczynników tłumienia,
model kontaktowy hertzowski i liniowy

In figure 10 the resulting vertical oscillations are depicted for several damping ratios.Obviously, the difference between the lines, corresponding to Hertzian contact force andto its linear approximations, is neglible.

It should be remarked that it is neither effective nor sensitive to solve non-hertzian contact problemsstep by step within dynamical simulations. The numerical cost is not acceptable, and the influence of the shape variation on the contact force is negligible.

7 Results of simulation

We start with a simple kinematical consideration. Assume that the center of a wheel moves at a prescribed constant traveling speed$V$.Further, assume that both bodies are rigid, and they are at all times in contact in a uniquely defined point of geometrical contact. Assuming a sufficiently large dry friction, at the point of contact there is no relative motion between wheel and rail, which amounts to the condition that the velocity of the contact point on the wheel circumference is zero.The above conditions define the trajectory of the wheel center and the revolution of the wheel uniquely.

Hence, we obtain vertical acceleration and angular moment and thus normal force and frictional force in tangential direction. We ought to underline thatunder such idealized conditions of contact, friction and rolling, we may calculate position and vertical force in the contact point without solving the equations of motion.

We derive from this a new type of traveling force problem:

$$F_y(x,t) = (F_0 + F_1 \sin(\Omega t))\delta(x - x_c(t)) \ , \tag{7}$$

where $x_c(t)$ is a relatively complicated function, the time derivative of which is shown in figure 11. By δ the Dirac distribution is denotated, and $\Omega = V/R_0$.

What is new here, is the oscillation of theconcentrated force position around its mean position, which moves steadily forward at speed V. Critical speeds and frequencies have been derived for the case of constant velocity. The oscillating position may exceed the critical speed, but only on small intervals. The effects of this need further considerations.

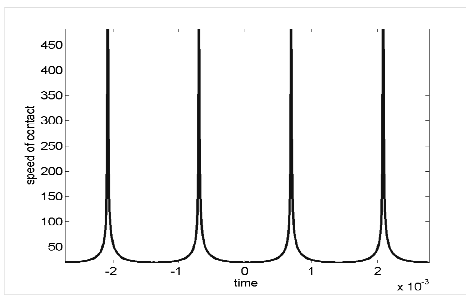

Fig. 11. Horizontal speed of contact point vs. time in the case of an average speed of 30 km/h [5]

Rys. 11. Prędkość pozycji wzdłużnej punktu kontaktu w przypadku szybkości średniej 30km/h [5]

Calculation of the accelerations resulting from the above assumptions on rigidity and ideal rolling conditions show that there has to be included a certain creepage as well as an elastic or visco-elastic approach of the contact partners and/or a compliance of the rail, e.g. due to a Winkler bedding. In that light, assumptions about constant horizontal speed of the wheel center or constant angular velocity of the revolving motion are not realistic.

In the result, models of a simplified bogie have been formulated, which feature a dynamics allowing for the mentioned effects. By solving the equations of motion, we map wheel imperfections to normal and frictional forces. These in turn drive the wear process, por. Fig. 8, upper part.

Analytical and numerical methods to integrate these factors into an update of the radius functions have been discussed in [12, 18, 3] and [2]. The model equations are:

$$y'(t) = f(t, y; u); \qquad y(0) = y_0 \ ; \tag{8}$$

$$u'(\varphi, t; y) = g\big(\varphi, t, u; y(t)\big); \qquad u(\varphi, 0; y_0) = u_0(\varphi) \tag{9}$$

for $0 < t < t_{fin}$.

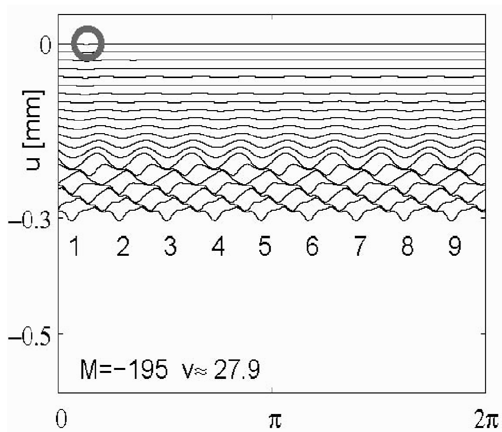

Fig. 12. Exemplary pattern evolution [18]

Rys. 12. Przykład rozwoju postaci poligonalizacji, por. [18]

In figure 12 a result from [18] is cited, which was obtained on the basis of the bogie model represented in figure 7. The sensitivity of the obtained pattern on the average speed of motion was discussed e.g. in [17].

8 Conclusions

For the contact of corrugated wheels and rails, we observe a considerable variation of the speed of the contact point around the horizontal speed of the wheel. The critical speed – from the point of view of the theory of traveling forces on supported beams – may be exceeded for very moderate speeds of wheel motion.

Above certain levels of expression of the corrugation patterns, lift off may occur, which is followed by an impact. The same is true in the case of poligonalized wheels, however, the critical speed is higher at the same amplitude of radius deviation.

Practically, a rigid constraint model is rather not realistic, viscoelastic models exhibit oscillations around the quasi-static trajectory. Initial deviations from the ideal constant wheel radius lead to variations of normal force, relative motion between the contact pair, and hence fluctuations of the friction force. These factors together drive the further evolution of the radius function. At certain traveling speeds, depending on loading conditions and compliance, polygonal patterns grow from any initial disturbance, e.g. a single trough or a flat spot.

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Summary

In the present paper the dynamics of a rolling railway wheel with imperfections, interacting with an ideal track, is investigated. For uniform guiding motion, the trajectory of the contact point and the resulting vertical accelerations are evaluated, first in the case of a rigid wheel, then in the case of elastic and visco-elastic contact. The resulting energy dissipation due to friction and percussive effects is calculated, and on that basis pattern formation is simulated.

Keywords: Railway mechanics, dynamics, corrugation

Dynamika koła kolejowego: korugacje, poligonalizacja i płaskie miejsca

Streszczenie

Praca dotyczy badania dynamiki koła kolejowego z nierównościami powierzchni tocznej oddziałującego z idealnym torem kolejowym (szyną). W przypadku ustalonego ruchu prostoliniowego określono trajektorię punktu kontaktu układu koło-szyna i wynikające przyspieszenia, początkowo przyjmując sztywne koło i szynę, a następnie rozważając przypadek kontaktu sprężystego i lepkosprężystego. Uwzględniając energię dysypowaną, określany jest ruch i ewolucyjna zmiana kształtu powierzchni tocznej koła.

Słowa kluczowe: mechanika kolejowa, dynamika, korugacja, poligonalizacja